On considère le polynôme

$$P(X) = X^4 + a_3X^3 + a_2X^2 + a_1X + a_0 \qquad ;$$

on note $\mathrm{F}_P(Y)$ le polynôme de Ferrari[1] associé.

On a

$$P(t) = \begin{bmatrix} t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{a_3}{2} & 0 \\ \frac{a_3}{2} & a_2 & \frac{a_1}{2} \\ 0 & \frac{a_1}{2} & a_0 \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \qquad .$$

On pose

$$C = \begin{bmatrix} 1 & \frac{a_3}{2} & 0 \\ \frac{a_3}{2} & a_2 & \frac{a_1}{2} \\ 0 & \frac{a_1}{2} & a_0 \end{bmatrix} \qquad ; \qquad D = \begin{bmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \end{bmatrix} \qquad .$$

On vérifie que l'on a

$$\mathrm{F}_P(Y) = 4\,\mathrm{dét}\,(C - YD) \qquad .$$

. . . .

[1] Si l'on a

$$P(X) = (X - \alpha_1)(X - \alpha_2)(X - \alpha_3)(X - \alpha_4) \qquad ,$$

alors on a

$$\mathrm{F}_P(Y) := (Y - (\alpha_1\alpha_2 + \alpha_3\alpha_4))(Y - (\alpha_1\alpha_3 + \alpha_2\alpha_4))(Y - (\alpha_1\alpha_4 + \alpha_2\alpha_3)) \qquad .$$